**MAGNIFICATION**

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# Magnification fundamentals

## A presentation of concepts and examples to clarify the subject of magnification

**by Bruce H.Walker**

The term *magnification* is frequently used (and often misused) when discussing the characteristics of an optical system. This presentation will attempt, through the use of discussion and examples, to clarify the true meaning of the term as it applies to a variety of optical systems.

### *Near object standard*

The term magnification generally relates to the apparent size of an object as it is appears to the human visual system. It follows that, in order to determine the magnification of an optical system, we must first establish a baseline, or a standard for comparison. In the case of near objects, (objects which are readily accessible) the accepted standard is the apparent size of that object when it is viewed at a distance of 254 mm from the eye. This distance is the established *near point* of vision, the closest that an object can be placed for comfortable, sustained viewing. While some individuals might not agree that this 254-mm standard applies in their particular case, the important point is that in order to facilitate meaningful comparisons a common starting point must be assumed. In discussing magnification of near objects, the 254-mm (10-inch) viewing distance *is* that accepted starting point.

It will be helpful as this discussion evolves to have a concrete example of a near object for reference in later paragraphs. To facilitate this, let's consider a U.S. quarter (25-cent piece), which is approximately 24 mm in diameter. When viewed at a distance of 254 mm with the unaided eye, this coin subtends an angle whose tangent is 24 ÷ 254, or 5.4 degrees (Figure 1).

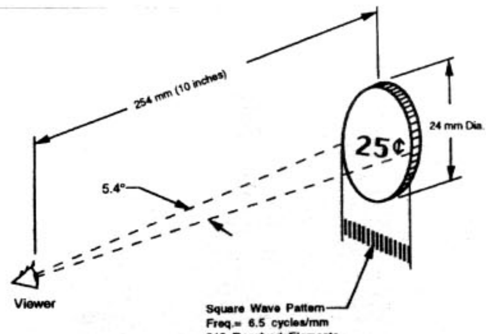


**Figure 1. The standard used in discussing magnification of near objects references the apparent angular size of an object as it appears when placed at the near point of the viewer's eye.**

For reasons that will soon become obvious, we will introduce the topic of visual resolution at this point. When we are able to view a repeating pattern of parallel black and white lines and distinguish the individual elements of that pattern, we say that the pattern has been resolved. At a distance of 254 mm, the normal eye can comfortably resolve a series of equal width black lines and white spaces with a frequency of 6.5 cycles/mm. If, as is shown in Figure 1, this 6.5 cycle/mm square wave target is superimposed on the 25-cent piece, it can be seen that it is possible to visually resolve 24 × 6.5 = 156 cycles, or 312 individual lines and spaces, across the 24-mm diameter. In other words, when viewing the 25-cent piece at a distance of 254 mm, we can resolve individual elements on the coin's surface that are as small as 24 ÷ 312 = 0.077 mm.

## *Loupe magnification*

Having established this standard for comparison, let's now consider a variety of optical systems and determine how they will affect the apparent size of the object (magnification) and potential resolution. The simplest optical device used for magnification purposes is the magnifier lens, or *loupe*. If we consider the loupe to be a thin lens held close to the eye, we find that, using the basic object image relationship shown in Figure 2, we will observe an image of the object which has been magnified by the following factor:

Magnification = (254 ÷ EFL) + 1

Where: EFL = focal length of the loupe in millimeters.

Therefore, for a loupe with a focal length of 25.4 mm, the magnification will be:

$$(254 \div 25.4) + 1 = 11\times$$

If we are viewing the referenced 25-cent piece with this loupe, the image will be $24 \times 11 = 264$ mm in diameter, located at a distance of 254 mm. Thus, it will now subtend a visual angle of about 55 degrees to the viewer's eye. Regarding the impact this magnification will have on resolution, we stated that it is possible for the normal eye to resolve 6.5 cycles/mm at the 254-mm viewing distance. Since the image formed by the loupe is at 254 mm, and the apparent size of the 25-cent piece has been increased to 264 mm, it follows that we will now be able to resolve

$6.5 \times 264 = 1{,}716$ cycles or 3,432 individual elements across the face of the coin, compared to 312 elements without the loupe. The size of the smallest resolved element has been reduced by the 11× magnification factor, from 0.077 mm down to 0.007 mm. In other words, the visual resolution capability, at the object, has been increased by the magnification factor from 6.5 cycles/mm to 71.5 cycles/mm. This, of course, is the basic reason for employing a magnifier to view a near object...to improve upon our ability to resolve detail in that object.

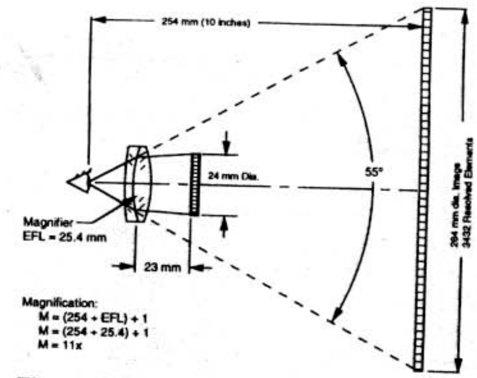


**Figure 2. A simple lens, or loupe, can be used as a magnifier to increase the apparent size of a near object as well as the amount of detail that can be resolved at the object.**

## *Microscope magnification*

When the limit of magnification for a simple loupe (about 20×) fails to reveal the desired level of detail, we find it necessary to apply a compound magnifier, i.e., a microscope system, to produce even greater magnification. Basic microscope optics provide magnification in the range from 10× to 600×. These values are achieved by incorporating two stages of magnification (Figure 3). In the microscope system, the objective lens forms a real, intermediate image of the object at magnifications that may range from 2× to 40×. The most common microscope objectives produce an intermediate image that is 10 to 18 mm in diameter, with that image located approximately 200 mm from the object.

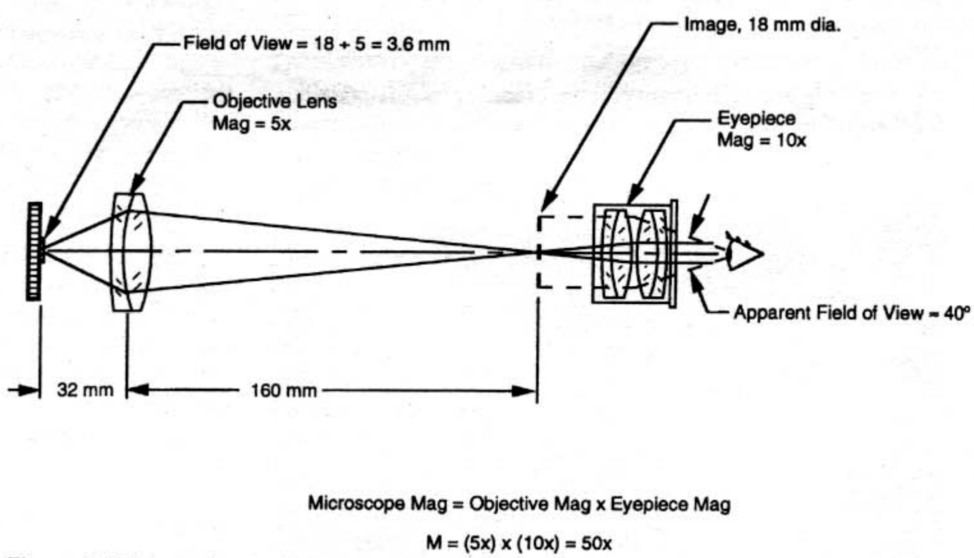


**Figure 3. When a simple lens cannot provide the required magnification, it is possible to introduce a system with two stages of magnification. Shown here is a basic microscope system with a total magnification of 50x.**

For example, in Figure 3 we have a 5× objective lens, forming an 18-mm diameter intermediate image. Based on standard optical calculations, we find that the object will be located about 32 mm from the objective (the working distance) and that the usable field of view will cover a spot on the object which is 18 ÷ 5 = 3.6 mm in diameter. Having created this magnified real image with the objective lens, the principles of loupe magnification can be applied to the microscope eyepiece to determine the final magnification of the microscope. In this example, using a 10× eyepiece, we can produce a final apparent image at a distance of 254 mm from the viewer's eye that is 50 times larger than the actual size of the object being viewed.

In the field of microscopy, resolution is generally specified in terms of the size of the smallest resolved element at the object. It was stated earlier that viewing the 25-cent piece with the unaided eye it is possible to resolve individual elements as small as 0.077 mm. Using the 50× microscope system we can reduce this resolved element size by the magnification factor to:

$0.077 \div 50 = 0.0015$ mm

per resolved element.

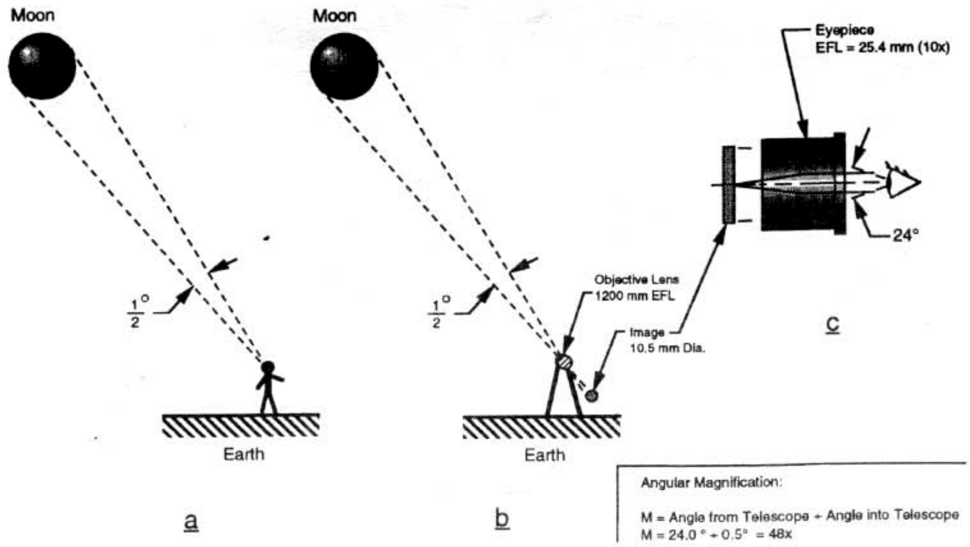


**Figure 4. (a) When considering magnification of a distant (as opposed to near) object, the moon is a convenient subject. (b) An objective lens with a focal length of 1200 mm will form an image of the moon that is 10.5 mm in diameter. (c) Viewing that 10.5 mm diameter image with a 10x eyepiece, the final image will subtend an angle of 24° to the eye.**

## *Distant objects*

Exploring the meaning of magnification for distant objects, (objects that cannot be placed at the 254-mm near point of the eye for examination), we encounter a considerably different set of conditions. For this example, we will now consider a distant object that is frequently viewed, magnified, and recorded, i.e., the moon (Figure 4).

The moon subtends an angle of approximately 1/2 degree when viewed from the earth with the unaided eye (Figure 4a). The most common optical instrument used for viewing the moon is the astronomical telescope. A typical amateur astronomer might have a telescope with a 1200-mm (48 inch) focal length objective lens. Such a lens will produce an aerial image of the moon at its focal plane that is 10.5 mm in diameter (Figure 4b). It might be possible (though not very useful) to view that image with the unaided eye from a distance of 254 mm. The angular subtense of that image to the eye would then be: $\tan^{-1} 10.5 \div 254 = 2.37°$. Without the objective lens, the moon subtends a 0.5° angle; with it, this angle is increased to 2.37°, a magnification factor of 4.74× due to the objective lens alone. In most cases we would choose to examine the aerial image formed by the objective lens with an eyepiece, thus creating a complete telescope.

An interesting phenomenon occurs when we use an eyepiece in this application. Knowing that the object being viewed is at a great distance, the viewer tends to focus the telescope such that the final image is located at infinity rather than the 254 mm distance which seems to be preferred when viewing near objects with a loupe or a microscope. This affects our eyepiece magnification formula slightly, simplifying it to:

$$Mag_{ep} = 254 \div EFL_{ep}$$

where $EFL_{ep}$ = eyepiece focal length in mm.

In determining the magnification factor of the objective lens alone we found it to be:

$$Mag_{ob} = EFL_{ob} \div 254$$

We may now multiply these two formulae to obtain the familiar telescope magnification formula:

$$Mag_T = Mag_{ob} \times Mag_{ep}$$

$$Mag_T = (EFL_{ob} \div 254) \times (254 \div EFL_{ep})$$

$$Mag_T = EFL_{ob} \div EFL_{ep}$$

Applying this formula for the case involving a 25.4-mm (10×) eyepiece, we can calculate the telescope's overall magnification as: $1200 \div 25.4 = 47\times$. Using this telescope, the moon will now subtend a quite impressive 24° angle to the eye (Figure 4c).

In discussing the effect of a telescope system on visual resolution it is most meaningful to consider angular resolution. The previously mentioned visual resolution of 6.5 cycles/mm at 254-mm distance leads to an angular visual resolution of 60 arc s per element (line or space) of the pattern being viewed. This angle will be reduced by the system magnification factor. Thus, for the 47× telescope, we will be able to visually resolve: 60 arc seconds ÷ 47× = 1.28 arc seconds/element at the object.

One final concept of optical magnification, applicable to distant objects, is that of *apparent viewing distance*. In the case of the moon, whose average distance from earth is about 226,000 miles, viewing through a 47× telescope we see the moon as it would appear to the unaided eye from 1/47th of that distance, or just 4800 miles. Thus, introduction of the telescope can be thought of as being equivalent to a 221,200 mile trip through space.

## *Television system*

When dealing with an optical system other than those where the object is viewed directly, the analysis and actual meaning of the term *magnification* becomes a little more complex. Let's consider a typical TV system, consisting of a 1/2-in. CCD camera, with a 12.5 mm objective lens, whose output is being viewed on a 19-inch monitor. Assume that the camera is viewing a 6-foot tall person, at a distance of 25 feet (Figure 5). That person will subtend an angle of 13.7 degrees to the camera lens, resulting in a 3-mm-high image on the TV camera sensor. For the standard 1/2-in. CCD camera format, the sensor is rectangular with a height of 3.6 mm. This means the image of the 6-foot tall person will cover $^{3}/_{3.6}$th of the total sensor height. On a 19 inch monitor the corresponding maximum picture height is 290 mm. Thus, the image height on the monitor will be $^{3}/_{3.6}$th of that, or 240 mm. The nature of the TV image structure dictates that, for optimum viewing comfort, the monitor should be viewed from a distance of about 4× the picture height or, in this case, 1160 mm. At that distance the 240-mm-high image will subtend an angle of 11.7 degrees. Since the original subtense of the subject to the camera lens was 13.7 degrees, we find that this system has an overall magnification that is less than 1:

Mag = 11.7° ÷ 13.7° = 0.85×
(sometimes referred to as minification).

## *Conclusion*

This paper, and the examples presented, have been intended to aide the reader in establishing a better understanding of the fundamental concepts of magnification and to assist in the communication of those concepts. The basic principles presented here are applicable to most commonly encountered optical systems.

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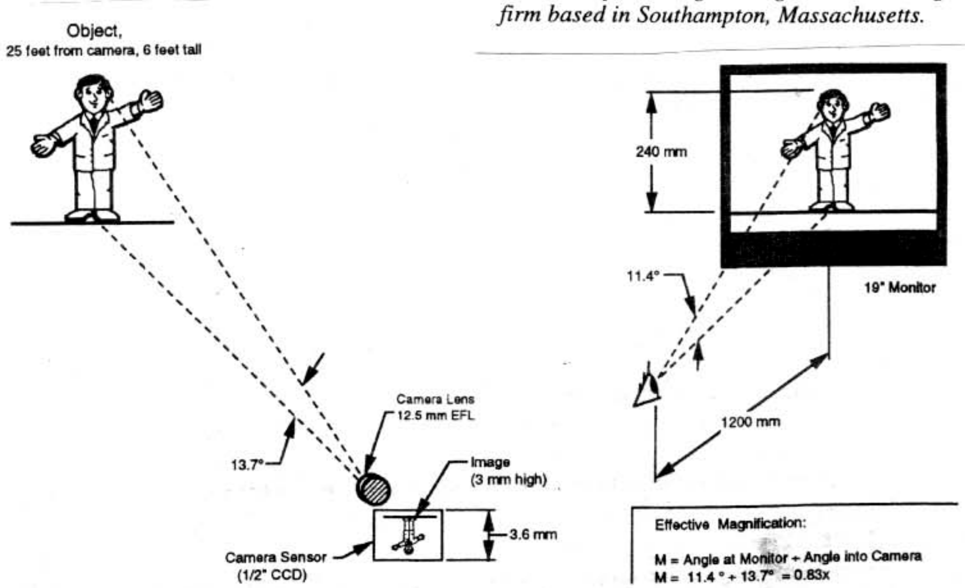


**Figure 5. The magnification of a television system is dependent upon the camera lens focal length, the camera sensor size, monitor size, and final viewing distance. All of these enter into the determination of effective system magnification.**

**WHEW!**

## MORE MAGNIFICATION

When you plug infinity into the object distance, it becomes impossible to calculate the magnification by the formulas. Here is the solution* to the question, how big is the image of the sun for a given focal length lens? (How crispy do you want that ant?)

4.55. In the Kitt Peak solar telescope a planar mirror 80 inches across tracks the sun, reflecting collimated light down a 500-foot shaft to a 60-inch parabolic mirror. This primary mirror, in turn, focuses the beam 300 feet back up the shaft where the image can be photographed. If the diameter of the sun is 864,000 miles and its distance from the earth is 93,000,000 miles, how large will its image be at the focus of the telescope?

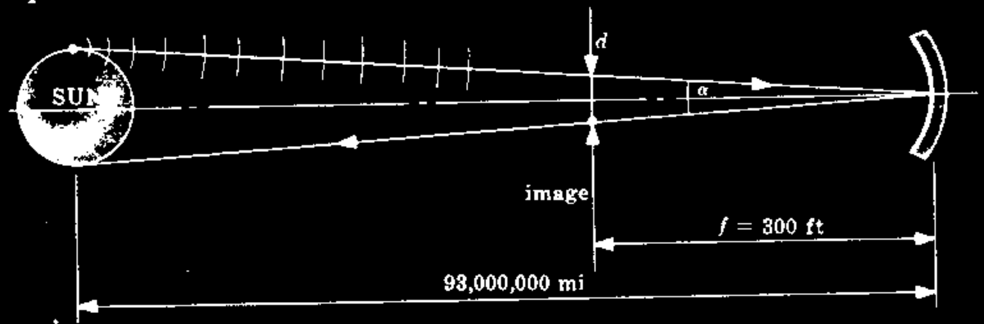


Fig. 4-47

Each point on the sun emits a spherical wave which increases in radius until it arrives and fills the aperture of a distant telescope with an almost planar wave. The nearly parallel bundle of rays is focused essentially to a point image a distance $f$ from the mirror. Thus, point by point, parallel bundles of rays entering at slightly different directions build up a complete inverted image of the sun. Of course, only the axial point on the sun will be perfectly imaged by a parabolic mirror, but the subtended angle ($\alpha$) is small and so there will be very little deterioration in the image over the entire disc. It follows from Fig. 4-47 that

$$\alpha = \frac{864{,}000 \text{ mi}}{93{,}000{,}000 \text{ mi}} = 0.0093 \text{ rad}$$

The diameter of the image disc is evidently given by

$$d = f\alpha = 300(0.0093) = 2.8 \text{ ft}$$

*Eugene Hecht, OPTICS, Schaum's Outline Series, McGraw-Hill Book Company, New York, 1975.