Optical Techniques For Increasing Image Width In Cylindrical Holographic Stereograms

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Optical techniques for increasing image width in cylindrical holographic stereograms

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Abstract

The image width in cylindrical holographic stereograms has been limited to about one-third the diameter of the display drum. An analysis of the optical system shows that producing an image which fills two-thirds of the drum diameter or more requires a cylindrical lens with an f/number of 0.5 or smaller. Cylindrical lenses of such small f/number and adequate optical quality are not commonly available. These optical characteristics can be achieved readily, however, with a holographic element on a curved surface. A system designed for increasing image width making use of such a holographic lens is described. The requirement for an extremely low f/number cylindrical lens can be eliminated by another technique which involves curving the holographic film over a short radius platen. This technique is also described. It is shown that both these techniques compensate the nonlinear horizontal magnification distortion in cylindrical holographic stereograms.

Introduction

In its usual configuration, the cylindrical holographic stereogram consists of a cylinder of holographic film illuminated by a conventional incandescent light bulb. A three-dimensional image appears at the center of the cylinder which displays motion when the cylinder is rotated or when the observer walks around the cylinder. This type of hologram, also known as the integral, lenticular, or multiplex hologram, was invented by Lloyd Cross in 1973.[1] The rainbow-colored action images of these holograms provide a fascinating and eyecatching display which has many applications such as use as a visual aid in education and as a point-of-purchase or exhibit display in advertising. To date, the multiplex hologram has exhibited several technical difficulties which have limited its exploitation in the marketplace. The images are rainbow colored, varying from red to blue as the vertical viewing angle is changed, and the vertical viewing range is quite limited. Time-smearing distortion limits the amount of motion which can be depicted in the hologram, and perspective distortion has limited the three-dimensional quality of the images. (A solution to this latter type of distortion has been demonstrated by Benton.[2]) Finally, it has been difficult to produce images which fill more than one-third the diameter of the display cylinder.

In this paper, we address the problem of increasing the image width in multiplex holograms. The optical system formed by the hologram and the viewer is analyzed to determine where the image is located and how its width is related to other geometrical optical parameters in the system. Two techniques for producing wide images in multiplex holograms are described. The first technique makes use of a curved holographic lens, and the second involves curving the holographic film over a short radius platen. Graphical plots and tabular data are presented that provide design information to aid in selecting optimal system parameters.

Analysis of image geometry

A description of the multiplex hologram process is presented in another paper by the authors.[3] Briefly, in this process a movie film transparency is imaged onto the hologram film in the vertical plane, and the projection rays are focused to a line on or near the film in the horizontal plane by a cylindrical lens. The horizontal exit pupil of the hologram is defined by the angle of the ray bundle emanating from this line focus in the horizontal plane. Because of the limited entrance pupil of the eye, only a narrow slice of this ray bundle is intercepted by the eye, and, therefore, only a narrow vertical slice of the image is viewed through the bundle. Sequential exposures and translation of the holographic film form a series of ray bundles emanating from line foci spatially separated by the amount of film translation. The entire width of the image is viewed through this lateral array of ray bundles. The image normally appears as a series of bright vertical bars separated by dark spaces associated with the exit pupil gaps between these ray bundles.

Figure 1 depicts two of these ray bundles and the rays associated with a given image point. For the purpose of this description and the analysis we present here, the simplifying assumptions are made that the cylindrical lens in the optical system has no aberrations and that a perfect line focus is formed on the surface of the hologram film.

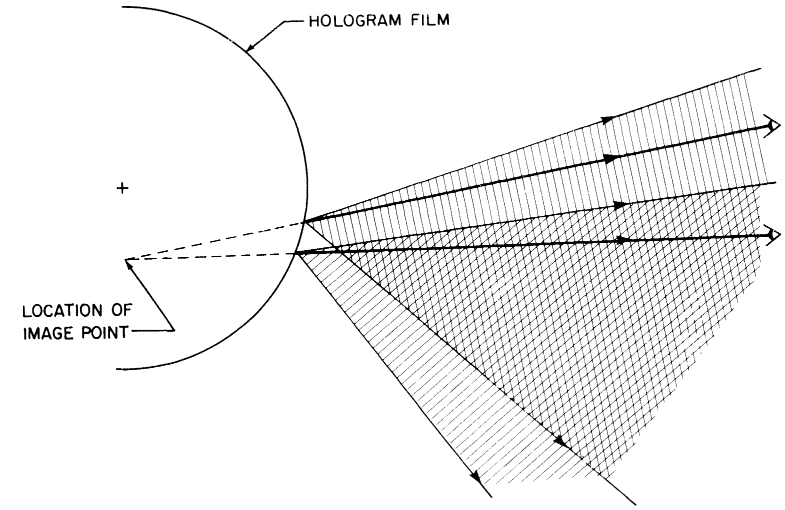

Figure 1. Multiplex hologram image ray geometry.

The mathematical location of a given image point is accomplished using the geometry shown in Figure 2. In this figure the two rays entering the eyes associated with image point P are shown by the bold lines and lie along lines l and l'. The image point we are locating here is a planar image point associated with a flat scene. This image would be produced by accomplishing the multiplex hologram process with a single motion picture film frame. Therefore, the image point P in the figure is not a stereoscopic image point perceived by the viewer with different perspective views for the two eyes. The location of image point P is determined by noting that the angle θ made by the image rays with film drum radial lines l_r and l'_r is the same for both eyes. The intersection of these two image rays, lines l and l', locates the image point. R_d is the radius of the hologram display drum, R_v is the viewing distance or the radius of the eye circle,

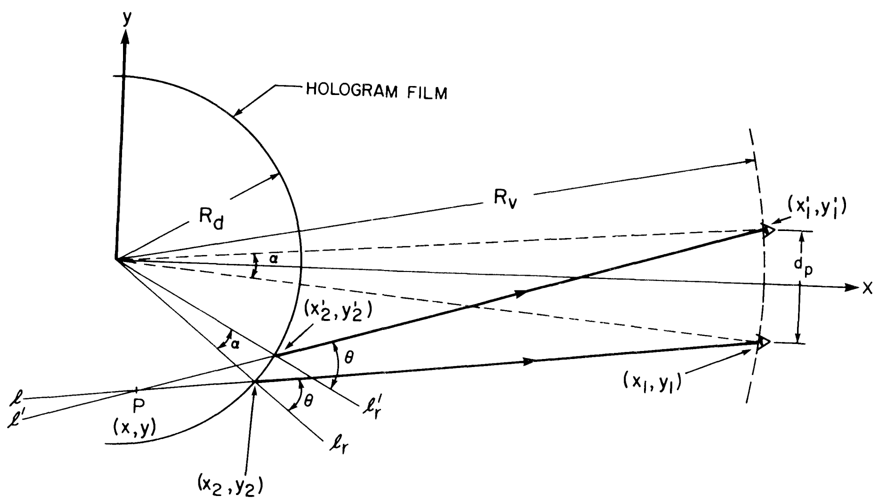

Figure 2. Geometry used to determine the location of image point P.

and the image point is defined by the coordinate point (x,y). The geometry of Figure 2 can be solved to give the following expressions for x and y for a given value of θ:

$$x = \frac{2\, R_v \sin \alpha/2 + \Gamma R_v \cos \alpha/2}{\Gamma} \tag{1}$$

$$y = \left[\frac{-R_d \sin \Psi + R_v \sin(\alpha/2)}{R_d \cos \Psi - R_v \cos(\alpha/2)}\right]\left[X - R_v \cos(\alpha/2)\right] - R_v \sin(\alpha/2) \tag{2}$$

where: $\Gamma = \dfrac{-R_d \sin \Psi + R_v \sin(\alpha/2)}{R_d \cos \Psi - R_v \cos(\alpha/2)} + \dfrac{R_d \sin \Psi' + R_v \sin(\alpha/2)}{R_d \cos \Psi' - R_v \cos(\alpha/2)}$

$$\psi = \gamma + \alpha/2$$

$$\psi' = \gamma - \alpha/2$$

$$\gamma = \theta - \sin^{-1}\left(\frac{R_d}{R_v} \sin \theta\right)$$

These expressions for x and y were computer evaluated for a film drum radius of 6 inches, a viewing distance of 36 inches, and an interpupillary spacing of 3 inches, and the results are plotted in Figures 3 and 4. In the presentation of these results we have taken the cylindrical lens and the projected image to be located a distance R_d

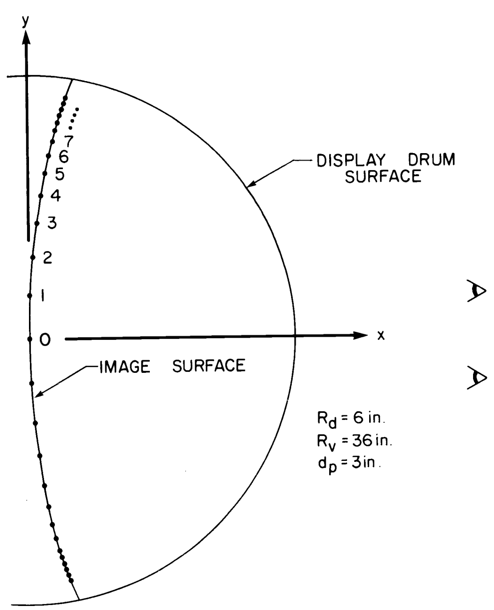

Figure 3. Cross sectional plot of the image surface.

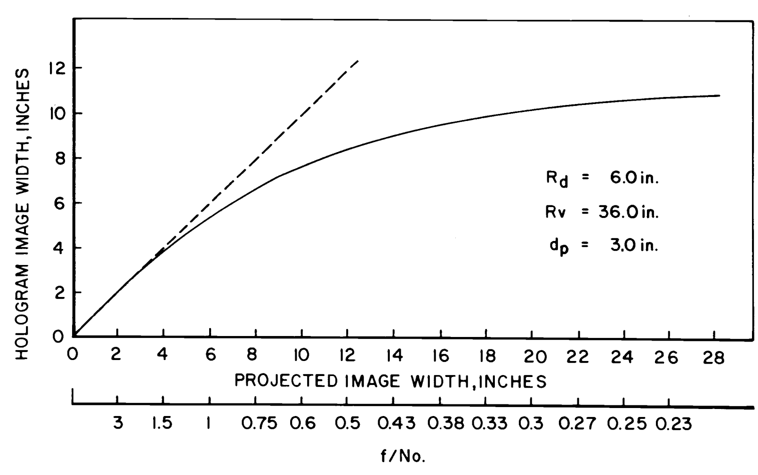

Figure 4. Plot of hologram image width as a function of projected image width and subject beam ray bundle f/No.

from the hologram film in the production of the multiplex hologram. In Figure 3 the central image point is seen to lie on the axis of the display cylinder, but other points lie off the diametral plane on a surface curved towards the viewer. It can be shown that this surface is circular with radius $R_V/2$. The numbered points on the image surface in Figure 3 correspond to equally spaced points in the projected image. The numbers give the distance of the projected image point in inches from the center of the image. These points are not linearly spaced along the hologram image surface and become bunched very closely together towards the edge of the drum. The cylindrical holographic stereogram process therefore results in a non-linear horizontal magnification distortion of the holographic image. From Figure 3 we observe that this distortion is quite small for an image width one-half the drum diameter or less. As the width of the image approaches the diameter of the drum, however, the distortion becomes quite severe.

In Figure 4 the width of the holographic image is plotted as a function of the projected image width and the f/number of the subject image ray bundle incident upon the hologram film. If the transformation of image points from the projected image to the holographic image were linear, all points on the plot in Figure 4 would lie along the dashed line. The deviation of the curve from this line is therefore a measure of the nonlinear horizontal magnification distortion. The plot in Figure 4 shows that to have the holographic image essentially fill the diameter of the display drum, an extremely wide projected image and, therefore, a very low f/number cylindrical lens is required. For example, to have a 10-inch-wide image in a 12-inch-diameter display drum would require a cylindrical lens with an f/number of about 0.3, assuming that the projection beam is collimated at the cylindrical lens.

Holographic lens technique

An f/number of 0.3 and adequate optical quality are not practical with a conventional cylindrical lens. The desired performance can be achieved, however, with a holographic optical element. A holographic cylindrical lens to produce wide stereogram images could be produced in a flat configuration as shown in Figure 5. An f/0.3 line source is probably not practical, but an f/0.3 holographic lens could be made by rotating a larger f/number source about its axis and multiply exposing the holographic film. Such a lens would produce a wide stereogram image but the nonlinear horizontal magnification distortion would remain. This distortion can be corrected, however, by producing the holographic lens on a curved surface as shown in Figure 6. The curvature of the required surface can be determined from the solution of the expressions for x and y above. These expressions relate the y coordinates of the holographic image and the projected image through the angle θ. From the computer solution of these expressions, the angle θ associated with holographic image points one through six was determined

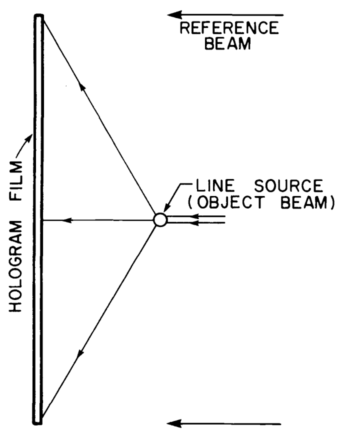

Figure 5. Scheme for producing a low f/No. holographic cylindrical lens.

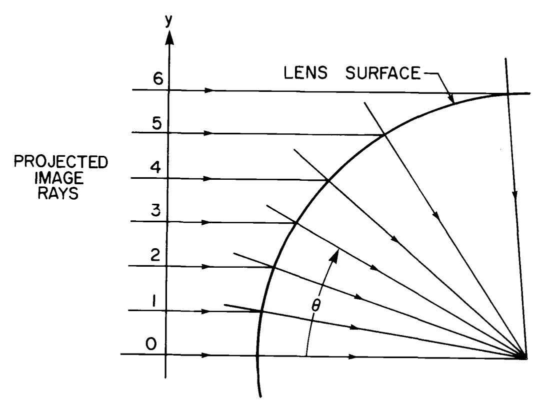

Figure 6. Holographic cylindrical lens on a curved surface.

and the rays were drawn in Figure 6. The intersection of projected image rays one through six and the focused image rays form a locus of points on the required lens surface. Not surprisingly, this surface turns out to be circular. This is fortunate since a circular shape simplifies the production of this holographic lens.

An on-axis holographic cylindrical lens configuration is shown in Figure 6. For this configuration the diffraction efficiency of the lens must be near 100 percent. If the diffraction efficiency of the lens is not high enough, the central vertical slice of the image will appear brighter than the remainder of the image. The off-axis lens configuration discussed in Reference 3 eliminates the need for high diffraction efficiency, but with a curved lens the off-axis configuration results in another type of image distortion shown in Figure 7. Therefore the on-axis configuration is preferred. Adequate diffraction efficiency should be achievable with dichromated gelatin recording material or silver halide film processed with one of the newer high-efficiency developing methods.[4]

Film curvature technique

Although we have not experimentally verified the technique, the use of a curved holographic cylindrical lens should produce wide, undistorted images in cylindrical holographic stereograms. The difficulties in making such a curved lens with sufficient diffraction efficiency may be objectionable, however, and limit its successful application. An alternate technique, which is simpler to implement, is to curve the holographic film over a short radius cylindrical platen, or film roller, in the multiplex exposure process. In this technique a conventional cylindrical lens or flat holographic cylindrical lens of reasonable f/number is used to form the line focus near the surface of the film. The optical configuration is shown in Figure 8. For a given projected image ray angle θ, the film incidence angle is δ. For a small diameter film roller, the angle δ can be much larger than the angle θ, and the associated hologram image point coordinate will be larger than that obtained when the film is flat and the film incidence angle is the same as the angle θ.

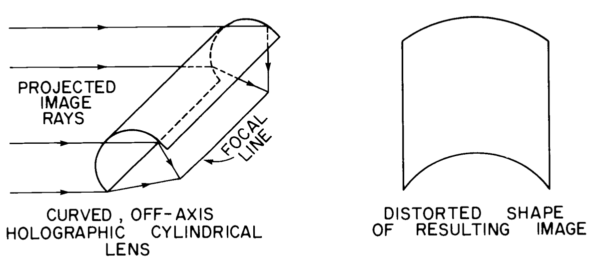

Figure 7. Image distortion with the use of a curved off-axis holographic cylindrical lens.

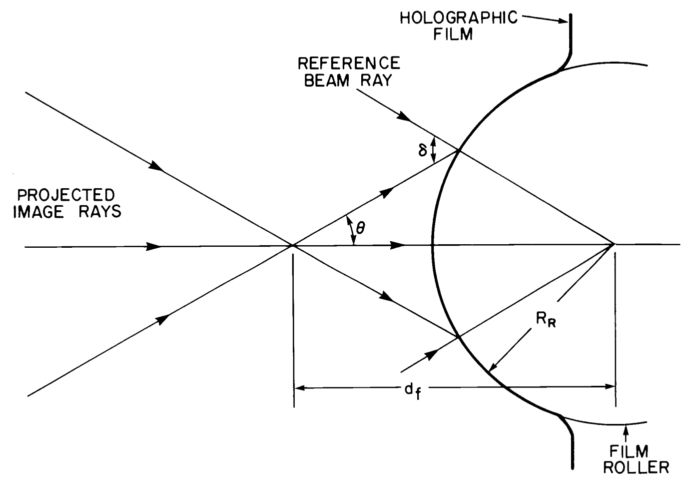

Figure 8. Optical system ray geometry with use of film roller.

The image point coordinates for the film roller geometry can be obtained by solving equations (1) and (2) for x and y and by noting that the angles δ and θ are related by the expression

$$\delta = \sin^{-1}\left(\frac{d_f}{R_r}\sin\theta\right) \tag{3}$$

where d_f and R_r are the quantities defined in Figure 8. In the solution of equations (1) and (2) for x and y for this geometry the angle θ is replaced by the angle δ as defined by equation (3). The solutions for this case are plotted in Figures 9 and 10. The image surface location and shape as shown in Figure 9 are the same as for the flat film case but the image point transformation is considerably different. In Figure 9 the image points are very nearly linearly located along the image surface. Thus, the film curvature technique also compensates for the nonlinear horizontal magnification distortion.

In Figure 10 the hologram image width obtained with the film curvature technique is plotted versus the width of the projected image and the f/number of the projected image ray bundle for various values of the ratio d_f/R_r. As d_f/R_r is increased, the plotted curve more nearly approaches the dashed line. When d_f/R_r equals one, the results are identical to that of the flat film case. As in Figure 4, the deviation of the plotted curve from the dashed line indicates the amount of nonlinear horizontal magnification distortion. From Figure 9 and 10 we observe that this distortion is quite small for values of d_f/R_r greater than 2. It is straightforward to show that the initial slope of the curves in Figure 10 (slope of the dashed line) is equal to the ratio d_f/R_r. This result provides a convenient mechanism for controlling the magnification of the image in the horizontal plane.

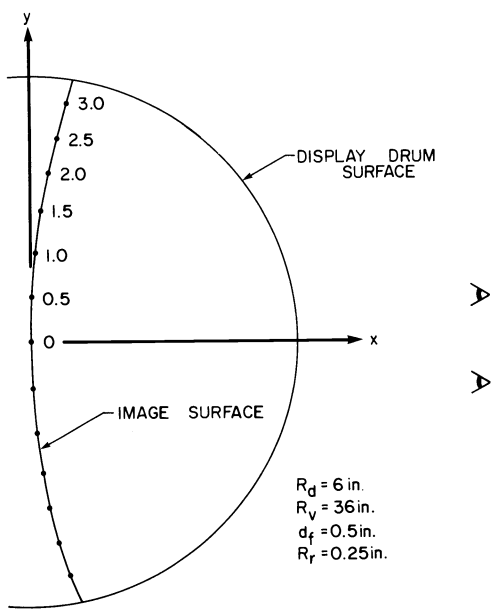

Figure 9. Cross sectional plot of the image surface obtained with the film curvature technique.

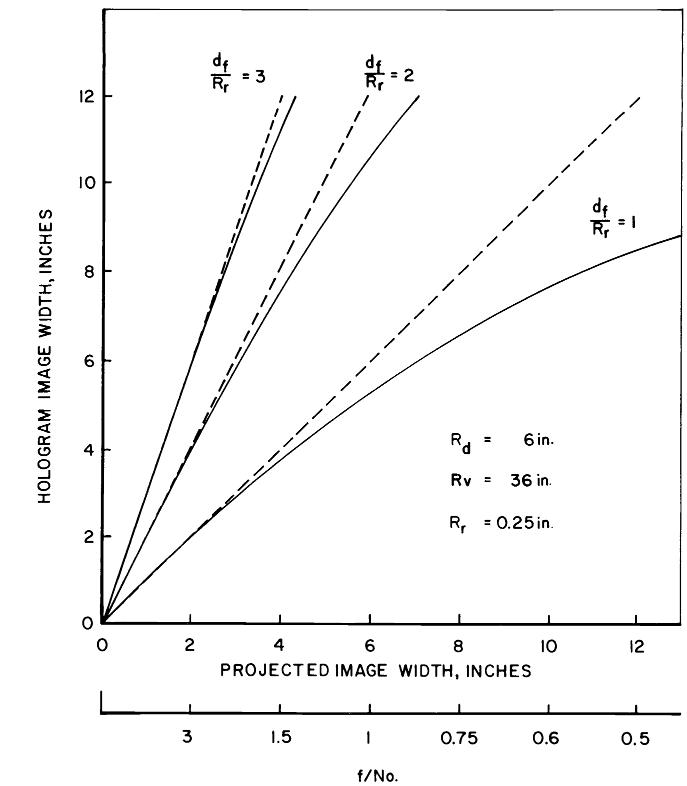

Figure 10. Plot of hologram image width versus projected image width and f/No. for various values of d_f.

Higher values of d_f/R_r yield higher horizontal image magnification and less nonlinear horizontal magnification distortion. Practical considerations set a limit to this ratio, however. The values of various optical system parameters are listed in Table 1 for several values of d_f/R_r.

Table 1. Film Curvature Optical System Parameter Values For Several Values Of d_f/R_r And a Hologram Image Width Of 10 Inches.
R_d = 6 in. R_v = 36 in.

d_f/R_r	f/No. (Projected Image)	f/No. (Reference Beam)	L_f (inches)
1	0.32	-	0
1.4	0.67	1.30	0.18
2	1.09	0.77	0.28
3	1.71	0.54	0.38

The values given are for a display drum radius of 6 inches, a viewing distance of 36 inches, and a hologram image width of 10 inches. L_f is the width of hologram film subtended by the subject beam. In the film curvature technique the reference beam must be focused to the center of the roller so that the reference beam rays are everywhere perpendicular to the film in the horizontal plane, thereby making the reference beam geometry the same as the illumination geometry in the display. As d_f/R_r increases, the f/number of the reference beam gets smaller, thereby making greater demands on the reference beam optics. In addition, as d_f/R_r increases, the width of film subtended by the projected image or subject beam rays increases, increasing the number of exposures required in the multiplexing process. Since image brightness declines with an increasing number of exposures, it is desirable to limit the number of exposures. The parameter values in Table 1 indicate that the optimal value for d_f/R_r probably lies between 1.5 and 2.

Conclusions

In this paper we have analyzed the cylindrical holographic stereogram to determine the location of the image points in the hologram and to determine how the width of the image is related to the optical system parameters. We have shown that the image width is limited by the f/number of the cylindrical lens used to focus the projected image rays onto the holographic film, and we have shown that use of a very low f/number lens with the hologram film flat results in nonlinear horizontal magnification distortion of the image. Techniques for producing wide images in holographic stereograms which compensate the nonlinear horizontal magnification distortion have been described. Both techniques presented here are capable of producing holographic stereogram images much wider than those obtained previously. The film curvature technique has been used by the authors to obtain an image width of 10 inches in a 12-inch-diameter display drum.

Acknowledgments

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